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THEORETICAL BASICS OF CONTROL ALGORITHMS DESIGN: EVALUATION OF DYNAMIC PROCESSES QUALITY

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ABSTRACT

Dynamic processes (or transients) quality is evaluated according to the system reaction on special tests signals. Among the tests signals, the most used is the input signal a.1(t), where 1(t)-unit function. For theoretical research, we conveniently use the unit function. Its actionon a dynamic element provokes a transient process h(t) called transient function. The transient function is constructed from the condition that the internal energy reserve of the system is zero. That is why the initial values of transient function are also equal to zero. In this paper we evaluate the transients quality

Keywords: *control algorithms design, quality evaluation, transients*

I. INTRODUCTION

Transients in an electric circuit occur whenever its steady state is changed. The cause of this change may be the operation of a switch, an open circuit, a short circuit, etc.

As in steady-state circuit analysis, transient analysis requires that all the circuit components be represented by their models or equivalent circuits made up of ideal resistive, capacitive and inductive elements, ideal voltage and current sources and switches.

Physically, the occurrence of transients can be traced back to the presence of inductances and capacitances in the affected circuit. The point is that the energy stored in the magnetic, and in the electric field, due to the inductive and capacitive elements cannot change instantaneously when it is switched.

The transient state of a circuit is described by differential equations, generally inhomogeneous. In the case of a linear circuit, the transient stateis described by linear differential equations, and in the case of a nonlinear circuit, by non linear differential equations.

An energy requirement of a rolling process is one of the highest among all technological processes in the metallurgical branch of industry. The rolling consists of rolling mills of different types, and such sufficient energy consumption can be explained by the fact that high power electric drives are used to rotate rolls of such mills. Even few percent reduction of these drives energy consumption will bring down prime cost significantly. This is especially actual for roughing mills. Automation plays a key role in this problem solving. The problem is that well-known P- and PI-control algorithms are widely used in automatic contol systems of the technological processes under consideration, as well as in 90-95% of all control loops in industry as a whole [1]. PI-controllers popularity can be explained by their structure simplicity and ease of use from staff point of view[2]. But, despite all the advantages, such controllers require a plant to be linear in order to keep demanded transient quality. This could be considered as a main disadvantage of such control algorithm, since a rolling mill is highly nonlinear, that is its parameters change during functioning [3]. That is caused by the following reasons: nonlinear electric drive characteristics, equipment deprication, rolling metal temperature, geometry, rolling speed are changed.

II. QUANTITATIVE CHARACTERISTICS OF TRANSIENTS QUALITY

Let us consider the quantitative characteristics of transient function through which we evaluate the dynamic processes quality.

Established value of transient function h(∞) – it is the value of transient function for infinite time. It corresponds to the value of transfer function w(0)=K_0, called the transfer static coefficient. The expression h(t)/h(∞) is the per unit transient characteristic $h^*(t)$.

In analyzing the transients quality, we often use the quantitative characteristics of rapidity and stability.

For *rapidity factors*, we have:

- Transient process duration : it is the time T_o from the moment of action of input signal to the time when output signal h(t) will differ from established value h(∞) not more than agiven small value Δ considered as acceptable error ;
- Delay time: it is the time T_d from the moment of action of input signal to the moment when the output signal is equal to half of its established value.
- Establishment time: it is the time T_{max} from the moment of action of input signal to the moment when the output signal reaches the maximal value h_{max} for the first time.

For *stability factors*, we have:

- Oscillations number of output expression around established value during the period of transient processT_0.
- Overshoot: the expression of augmentation σ of established value by the ordinate of oscillation transient function. It is usually expressed in percents. For that purpose, we consider the established value as basic value.

The illustration of quality factors for per unit transient function h$(t)^*$=h(t)/h(∞) is shown on figure 1.

III. ETALON TRANSIENT AND TRANSFER FUNCTIONS, CHARACTERISTIC POLYNOM

We have certain constraints related to transient process quality. We consider that transient function has good quality factors if the overshoot value σ is not above 0,05, and the duration of transient process is minimal.

We examine the set of transient functions that satisfy given constraints. They are considered as etalon or hopeful functions. The transient function h(t) corresponds to Laplace representation W(P)/P, where W(P) is the transfer function. Etalon transient function can be presented in relation with the hopeful transfer function

$$W_h\,(P) = K/C_n(P) \qquad (1)$$

Where$C_n(P)$ – etalon (or hopeful) characteristicpolynom; K- static transfer coefficient. It characterises the scale of output signal.

If K=1, then we have the reglemented transfer function

$$W_h(P) = 1/C_n(P)$$

The characteristic polynom can be expressed as follows:

$$C_n(P) = (T.P)^n + d_{n-1}(T.P)^{n-1} + \cdots + d_1(TP) + 1$$

Called standard polynom, with T- middle geometric time constant;$d_1, d_2, \ldots, d_{n-1}$ - non dimensional coefficients.

The middle geometric time constant T characterises the scale of transient function. If we note TP=S, then we have the transfer function:

$$W_h(S) = 1/C_n(S)$$

With $C_n(S) = S^n + d_{n-1}S^{n-1} + \cdots + d_1 S + 1$ that is the reglemented characteristic polynom.
The nondimension coefficients will characterise the form of transient function.

IV. ETALON APERIODICAL TRANSIENT CHARACTERISTICS[4]

If the overshoot is not acceptable, then as etalon transfer function, we can use an element whose roots are real negative numbers. Reglemented characteristic polynom in that case can be presented in the following state:

$$C_n(S) = S^n + d_{n-1}S^{n-1} + \cdots + d_1 S + 1 = (S + S_1)(S + S_2)\ldots(S + S_n)$$

Where $S_{1,}S_{2,} \ldots\ S_n$ – roots of characteristic equation satisfying the condition $S_1 S_2 \ldots \ldots S_n = 1$.

If we neglect all the components of reglemented characteristic polynom of first power:

$$C_n(S) = T_\Sigma . S + 1$$

Where $T_\Sigma = (1/S_1 + 1/S_2 + \cdots + 1/S_n)$- time constant that characterises the duration of transient process.

Considering that $S_1 S_2 \ldots S_n = 1$, obviously the minimal value of T_Σ will match with $S_1 = S_2 = S_2 = \cdots = S_n = 1$.

Thus, we have the following etalon characteristic polynom:

$$C_n(S) = (S + 1)^n = S^n + d_{n-1}S^{n-1} + \cdots + d_1 S + 1$$

That polynom is called ''newtownbinom'' and non-dimension coefficients $d_{1,} \ldots, d_{n-1}$ are called ''coefficients of newtownbinom''. Characteristicpolynomsthat are Newtown binom look as follows:

$$S^2 + 2S + 1;$$
$$S^3 + 3S^2 + 3S + 1;$$
$$S^4 + 4S^3 + 6S^2 + 4S + 1; \ldots$$

The reglemented characteristicpolynom of order n corresponds to the aperiodical transfer function:

$$W_h(S) = 1/(S + 1)^n \qquad (2)$$

Transient functions that are created by the etalon reglemented transfer function (2) are represented in figure 2.

The relation (2) can be approximated by aperiodical elements of first order:

$$W_h(S) = 1/(S + 1)^n \approx 1/(nS + 1)$$

The plots of transient characteristics of aperiodical element of second order and its approximation with element of first order are shown in figure 3.

If we fix the precision of establishment of transient process equal to $\Delta = 0.05$, then the duration of transient for aperiodical element of first order will be almost equal to 3T.

V. ETALON OSCILLATING TRANSIENT CHARACTERISTICS[7][8]

If we consider that transient characteristichas the overshoot σ that is less than 0.05, then the transient process duration will be smaller than the one of etalon aperiodical transient process. We have the case of characteristic equation whose roots are:

$$S_1 = \delta + j\omega\,, \qquad S_2 = \delta - j\omega$$

This couple of roots corresponds to reglemented characteristic polynom of second order:
$C_2(S) = (S + S_1).(S + S_2) = S^2 + 2\delta S + 1 = S^2 + dS + 1,$

where $|S_1| = |S_2| = (\delta^2 + \omega^2)^{1/2} = 1$ – roots modules, d=2δ.

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Reglemented characteristic polynom corresponds to the transfer function

$$w(S) = 1/(S^2 + 2\delta s + 1) \qquad (3)$$

That induces the transient function:

$$h(t/T)^* = 1 - \left(cos\frac{\omega t}{T} + \frac{\delta}{\omega}.sin\frac{\omega t}{T}\right)\exp\left(-\frac{\delta t}{T}\right)$$

The maximal value of transient function is reached at the time $t = Tmax = \pi t/\omega$.The function has relative overshoot:$\sigma = h(Tmax/T)^* - 1 = exp\ (-\pi\delta/\sqrt{1-\delta^2})$.

From that last expression, σ=0.05 corresponds to $\delta = 0.69$, (d=1.38) and established time $T_{max} = 4.38T$.

Thus etalon reglemented characteristic polynoms

$C_n(S) = (S + 1).(S^2 + 2\delta S + 1)^{(n-1)/2}$for n= 1,3,... ;
and$C_n(S) = (S^2 + 2\delta S + 1)^{n/2}$ for n= 2,4,… ; (4)

The real parts of roots δ is chosen to ensure overshoot$\sigma = 0.05$. Reglemented etalon characteristic polynoms with equal roots for $\sigma = 0.05$ will have the aspect represented in table 1.

Table 1. Polynom

n	δ	T_{max}	Полином $c_n(s)$ (6.11)
2	0,690	4,34	$s^2 + 1{,}38 \cdot s + 1$
3	0,573	5,28	$s^3 + 2{,}14 \cdot s^2 + 2{,}14 \cdot s + 1$
4	0,729	6,54	$s^4 + 2{,}92 \cdot s^3 + 4{,}13 \cdot s^2 + 2{,}92 \cdot s + 1$
5	0,666	7,43	$s^5 + 3{,}66 \cdot s^4 + 6{,}44 \cdot s^3 + 6{,}44 \cdot s^2 + 3{,}66 \cdot s + 1$
6	0,741	8,58	$s^6 + 4{,}45 \cdot s^5 + 9{,}59 \cdot s^4 + 12{,}15 \cdot s^3 + 9{,}59 \cdot s^2 + 4{,}45 \cdot s + 1$

The etalon transient characteristics that correspond to those polynoms are represented on figure 4a.

The transient process created by the element of order n is damped faster when we consider that the modules of complex roots of characteristic polynom are not mutually equal, and they have the following distribution:

$$S_K = \widetilde{S_K} = |S_K|^2 = 2^{(m/2+1/2-K)};$$

Where $\widetilde{S_K}$ – complex conjugate of S_K , K= 1,2,… ;

$$m = floor\ (n/2);\ |S_1||S_2| \ldots |S_n| = 1.$$

Thus etalon reglemented characteristic polynoms:

$$\begin{cases} And \quad C_n(S) = (S+1)\left(S^2 + 2\delta S + 2^{\left(\frac{m}{2}+\frac{1}{2}-K\right)}\right)^{\frac{n-1}{2}}\ for\ n = 1{,}3\ \ldots; \\ C_n(S) = (S^2 + 2\delta S + 2^{(m/2+1/2-K)})^{n/2}\ for\ n = 2{,}4, \ldots; \end{cases} \qquad (5)$$

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Table 2 . Polynom

n	δ	T_{max}	Полином $c_n(s)$ (6.12)
2	0,690	4,34	$s^2+1{,}38\cdot s+1$
3	0,573	5,28	$s^3+2{,}14\cdot s^2+2{,}14\cdot s+1$
4	0,663	6,22	$s^4+2{,}65\cdot s^3+3{,}88\cdot s^2+2{,}81\cdot s+1$
5	0,604	7,16	$s^5+3{,}42\cdot s^4+5{,}99\cdot s^3+6{,}13\cdot s^2+3{,}56\cdot s+1$
6	0,587	7,75	$s^6+3{,}52\cdot s^5+7{,}62\cdot s^4+9{,}81\cdot s^3+8{,}31\cdot s^2+4{,}10\cdot s+1$

The real part of roots δ is chosen so as to ensure overshoot $\sigma = 0.05$. Reglemented etalon characteristic polynoms with distributed roots and $\sigma = 0.05$ will have the aspects shown in table 2.

The corresponding etalon transient functions are shown on figure 4.b.

We often use the oscillating element of second order as hopeful transfer function:

$$W_h(P) = 1/(2T_\mu{}^2.P^2 + 2.T_\mu P + 1) \qquad (6)$$

This element has the damping coefficient$d = \sqrt{2} \approx 0.707$,

Overshoot$-\sigma = 0.043$; establishment time $T_{max} = 6.28T_\mu$.

The imaginary part and the real part of characteristic polynom roots of that transfer function are equal to $1/T_\mu$.

VI. ETALON MATRIX DIFFERENTIAL EQUATIONS[5][6]

Control objects dynamics can be described by a system of differential equations having high order. Apart from that, a control system can have many inputs and outputs related one to the other. Such systems are as a rule described by matrix differential equations:

$$PX = A.X + B.U$$

Where X- vector of state variables with dimension n ;
U- vector of control signals with dimension m ;
A- vector of system parameters with dimension $(n * n)$;
B- vector of system parameters with dimension $(n * m)$;
$P = d \dots/dt$.

The character of transient depends on the matrix A parameters. The characteristic polynom of differential equations system is:

$$C_n(S) = \det(S.1 - A.T),$$

Where T- is middle geometric time constant.

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The roots of characteristic polynom are $S_1, S_2, \ldots, S_n$.Theydetermine the character of transient processes. They are also called the proper values of the matrix. We determine the parameters of Matrix A that correspond to the hopeful characteristic polynom.

Let us consider a matrix of second order:

$$A_{2h} = -\frac{1}{\sqrt{2.T}}(1+E) = -\frac{1}{\sqrt{2.T}} \cdot \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Where 1= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

That matrix corresponds to etalon reglemented characteristic polynom of second order:

$$\det(S.1 - A_{2h}.T) = S^2 + \sqrt{2}S + 1$$

Etalon matrices of second, third and next orders can be defined as Jordan diagonal matrices:

$$A_{2h}.T = diag\ (1 + E.a)\ for\ a = 1.050;$$
$$A_{3h}.T = diag\ (a.1, 1 + Ea)\ for\ a = 1.630;$$
$$A_{4h}.T = diag\ (1 + E.a, 1 + E.2a) for\ a = 0.785;$$
$$A_{5h}.T = diag\ (a.1, 1 + E.a, 1 + E.2a)\ for\ a = 1.400;$$
$$A_{6h}.T = diag\ (1 + E.a, 1 + E.2a, 1 + E.3a)\ for\ a = 0.656\ etc.$$

The etalon characteristic polynom of $n - order$will look as follows:

$$\det(S.1 - A_{nh}.T),$$

Where T- is time constant that is a characteristic of the scale. Reglemented characteristic polynoms correspond to etalon matrices (Table 3).

Table 3 . Polynom

n	δ	T_{max}	Полином
2	1,050	4,34	$s^2 + 1{,}38 \cdot s + 1$
3	1,630	5,01	$s^3 + 2{,}01 \cdot s^2 + 2{,}11 \cdot s + 1$
4	0,785	6,17	$s^4 + 2{,}60 \cdot s^3 + 3{,}84 \cdot s^2 + 2{,}80 \cdot s + 1$
5	1,400	6,47	$s^5 + 2{,}63 \cdot s^4 + 5{,}07 \cdot s^3 + 5{,}27 \cdot s^2 + 3{,}32 \cdot s + 1$
6	0,656	7,82	$s^6 + 3{,}17 \cdot s^5 + 7{,}88 \cdot s^4 + 10{,}13 \cdot s^3 + 8{,}47 \cdot s^2 + 4{,}15 \cdot s + 1$

The etalon matrix system of second order can be represented as follows:

$$T^2.P^2.X + d.T.PX + X = B.U,$$

Where d- damping parameter;

P=S/T. in that case the elements of vector X have independent transient functions of second order.

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VII. CONCLUSIONS

Transients quality is often evaluated through the transient characteristic. It is considered that transient characteristic has good quality factors if the overshoot value is less than $\sigma = 0.05$, and with minimal duration of transient process.

Transfer functions that create such transient characteristics are called etalon. Very often as etalon transfer function we use oscillating element of second order (6). For the design of system with high order of transfer function, it is advisable to use etalon transfer function having etalon characteristic polynoms (4) or (5)./.

Figures:

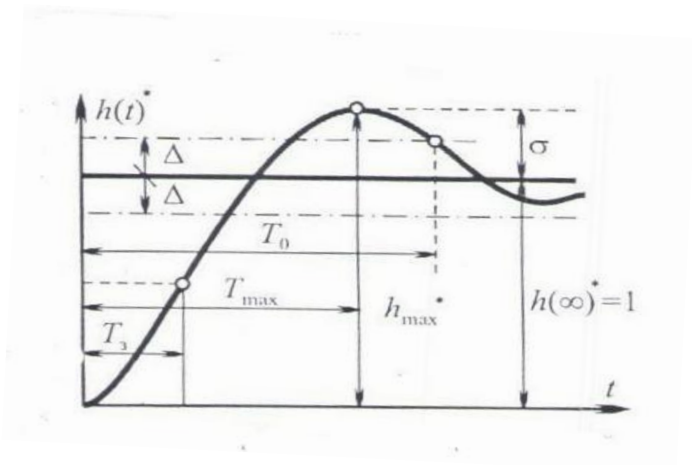

Figure 1 : Quality indicators of transient function

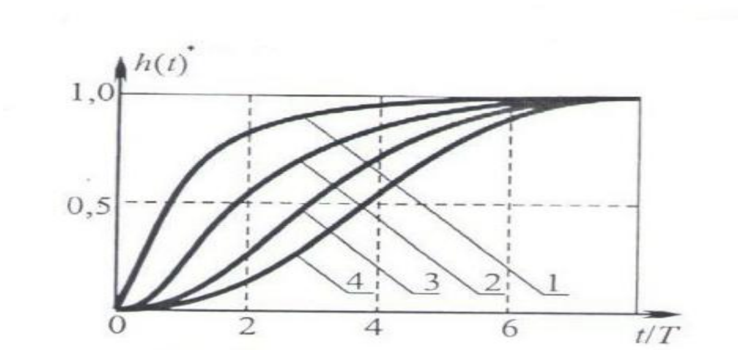

Figure 2 : Transient characteristics created by aperiodical elements of order n = 1,2,3,…

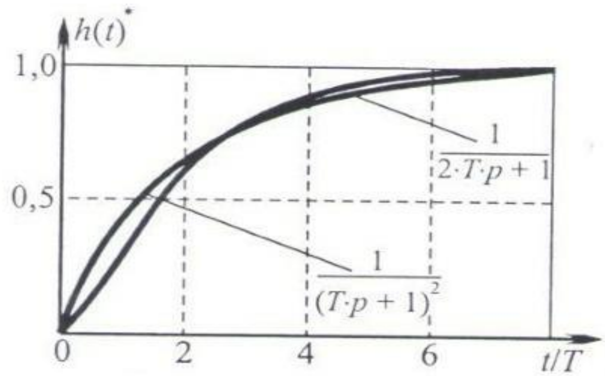

Figure 3 : Transient characteristics of a periodical elements for first and second order

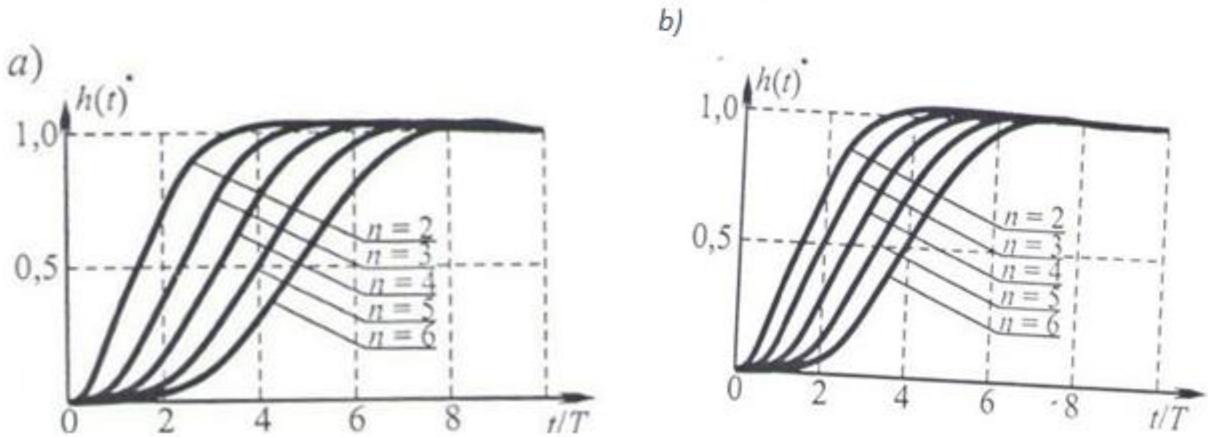

figure4: Etalon transient functions of order n = 2,3,4,5,6
a) For equal values of roots b) For distributed roots modules

.

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